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DECOUPLING EFFECT OF MULTI-STEPPED BI-MESA AT-CUT QUARTZ RESONATORS

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Abstract - **In this paper, we calculated the mode-coupling characteristics of multi-stepped bi-mesa resonators and compared them with beveled resonators. Calculation results showed that increasing the number of mesa steps reduced coupling between the thickness-shear and thickness-flexure modes. These results indicate that the multi-stepped bi-mesa resonator has a good mode-decoupling effect, and its performance characteristics approach those of the beveled resonator.**
***Keywords* - Mesa, multi-stepped mesa, AT-cut, quartz, resonator**

I. INTRODUCTION

To study the effect of multi-stepped bi-mesa structures on the decoupling of thickness-shear (TS) and thickness-flexure (TF) modes, we calculated the mode-coupling characteristics of multi-stepped bi-mesa AT-cut quartz resonators. The TF mode couples with the TS mode strongly because the TS mode contains the TF components[1,2]. Hence, the temperature characteristics, crystal impedance, and Q value of the fundamental TS mode are strongly affected by the components of the TF.

To avoid these couplings, contouring or mesa structures can be applied, which provide greater energy trapping effect than mass-loading of the electrodes[3]. Over the past few years, there has been an increased demand for miniature resonators. When requirements call for a small resonator, contouring one precisely is difficult because a mechanical grinding process must be used. On the other hand, mesa structures are fabricated by using a chemical etching process, therefore, they have an advantage in that they can be fabricated precisely to realize small resonators. We have shown previously that mesa and stepped mesa structures have better decoupling effects than mass-loading of the electrodes[4-6]. However, multi-stepped mesa structures comprised of more than three mesa steps have not been studied yet.

In this paper, we calculated multi-stepped bi-mesa resonators and compared them with beveled resonators. A two-dimensional finite element method was used for the calculations in the X-Y' plane. Calculation results showed that increasing the number of mesa steps reduced coupling between the TF and TS modes. These results indicate that the multi-stepped bi-mesa resonator has a good mode-decoupling effect, and its performance characteristics approach those of the beveled resonator.

II. CALCULATION MODELES

Figure 1 shows 2-dimensional models of a multi-stepped bi-mesa and a beveled resonator. A two-dimensional finite element method (2D-FEM) was used to perform the calculations in the X-Y' plane. The thickness-to-width ratio (a/b) of the resonator was set in the range of 20 to 30, and the ratio of the thick area (e/b) was set at 15. To compare the multi-stepped resonator with a beveled resonator, the stepped region was slanted from the thick area to the edge of width, which was the same as the beveled slope as shown in Fig. 1(b). Triangle elements were used to solve the beveled resonator, and only the region of the plate where $0 \leqq x1$ and $0 \leqq x2$ needs to be considered since the result can be extended due to symmetry. The thickness of the plate edge was changed from 99% to 1% of the thick area. The total number of mesa steps was changed from 1 to 10.

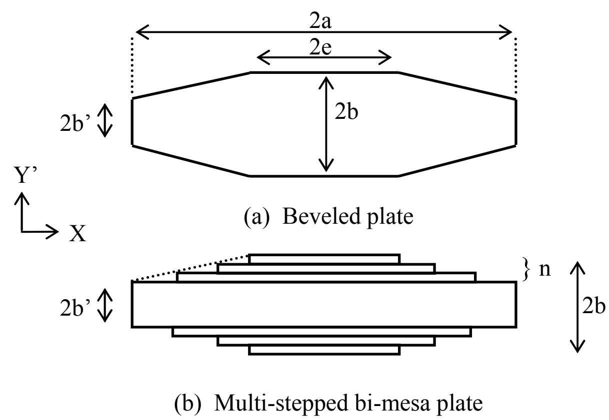

Fig. 1. Schematics of quartz plates

Figure 2 shows the fundamental thickness-shear (TS-1) displacement of the flat plate. The displacement u2 indicates that the flexural component is excited by shear deformation because flexural motion is present in shear deformation. Therefore u1 displacement also contains the same frequency component (phase difference is 90 degrees). To evaluate this mode coupling strength, the minimum relative frequency difference between the TS-1 and spurious modes are usually estimated from the mode spectrum[2]. This frequency difference can be estimated between TS-1 and targeted spurious modes independently. However, a number of

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calculations are needed to estimate the minimum value of the frequency difference.

III. Mode Coupling Estimation Using DFT

In this work, the mode-coupling strength between the TS-1 and the coupled TF mode were estimated using discrete Fourier transform (DFT) to reduce the calculation time. The DFTs were performed on the u1 displacement of the surface in the thick region, and a Rosenfield window function was used to survey weak frequency components. The major frequency component is 0 because a typical TS-1 displacement locates over the y = 0 axis (average > 0) in the thick region. This 0 frequency component masks the component of the pure thickness-share mode near 1. To prevent this masking, the average displacement was subtracted from the total displacement.

Figure 3 shows the frequency components of the flat and bi-mesa plates. The frequency component near 2 was classified as TS-1 frequency, and coupled TF component was near 9. The value of the TF components of the bi-mesa plate, which is lower value than the flat plate, shows the mode-coupling suppression of the large energy-trapping effect. To evaluate this mode-coupling suppression, $H(k_{tf})/H(k_{ts})$ was used as a coupling strength.

Figure 4 shows the estimated mode-coupling characteristics for the bi-mesa plates. The ordinate is the mode-coupling strength, the abscissa is the height of the thin region, and each plate has a different length-to-thickness ratio a/b (20, 25, 30). The mode-coupling strength for all plates decreased gradually as b'/b decreased from 1.00 to 0.85. Increasing the mesa height (b-b') increased the strength of the energy trapping effect decreasing the coupling between the TS and TF modes, and all curves had minimum values. As the mesa height was further increased, the thick region eventually dominated the overall geometry of the plate. In the lower range of b'/b = 0.5, the plate acted more like a plate without lower regions and the edge of the mesa region began to contribute more strongly to mode-coupling than the edge of the plate. However, there are some minimal values in the range of b'/b = 0.7 to 0.1. For the a/b = 20 plate, two minimum points are present, b'/b = 85.3 and 69.0 whose displacements are shown in Fig. 5.

In the thick region, both u1 displacements are clear cosine curves containing uncoupled TF components and look to have almost the same shape. In the outer region, each displacement is different. For the b'/b = 85.3, the amplitude of the u2 displacement is one-third of that for b'/b = 69.0. If the displacement of the plate edge was larger, energy loss, which occurred by a mounting, increased. Keeping the displacement of the edge low is preferable for any resonators. In addition, if one mesa step height was very high, the face-shear mode in the X-Z' plane would couple with the TS-1 mode in practice. Therefore, choosing the minimal points in high mesa step height offers less advantage.

At the minimal point, the TS and TF displacements separated into the thick and outer regions respectively. This is because the minimal points fulfill the boundary condition trapping the TF mode only in the outer region. Decreasing the b'/b increases the TF frequency in the outer region slightly, therefore, minimal points exist periodically.

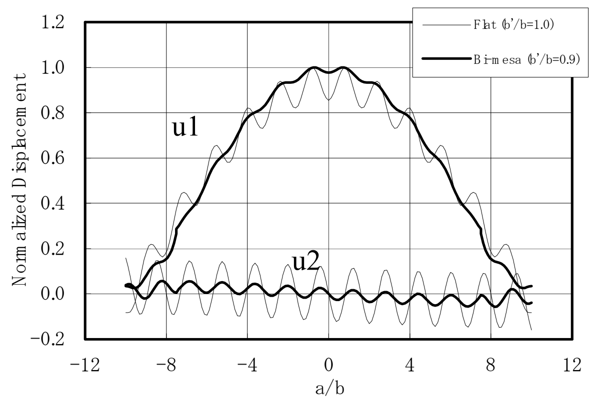


Fig. 2. Fundamental TS-1 displacements (a/b=20.0)

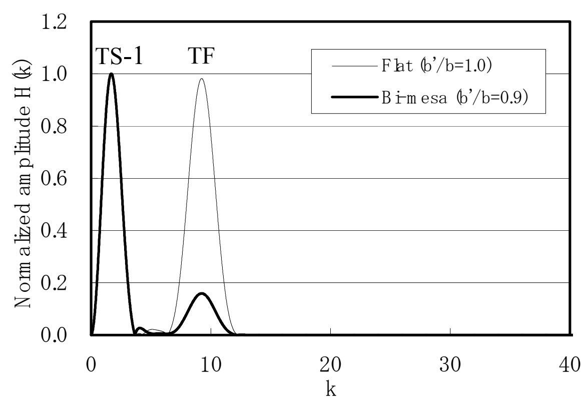


Fig. 3. Frequency spectrum of u1 displacement in thick region

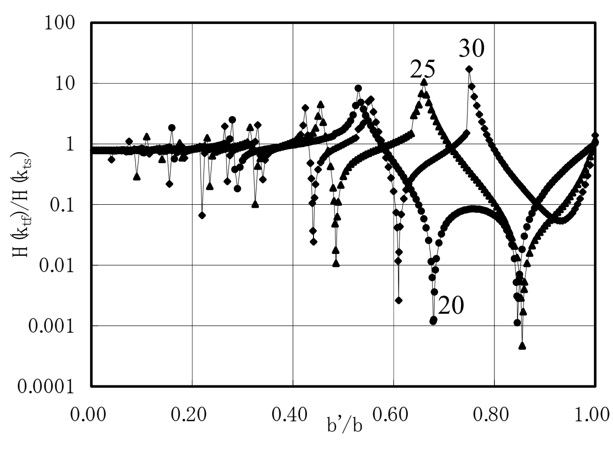


Fig. 4. Coupling-strength for bi-mesa (1-stepped) plate (Numbers indicate values of a/b)

IV. Beveled And Multi-stepped Bi-mesa Plates

The mode-coupling characteristics of the beveled and multi-stepped bi-mesa plates are shown in Figure 6. For the beveled plates, minimum values of a/b = 20.0, 25.0 are 1.46×10^{-4} at b'/b = 0.499 and 1.97×10^{-4} at 0.816, respectively, and for a/b = 25, mode-coupling strength remained at 3.4×10^{-2} in the wide range, however, this value is not so good compared with other a/b ratios.

For the multi-stepped plates, the minimum coupling-strength values were not so different from that of the beveled plates. Figure 7 shows the minimum values near b'/b = 1.0 and their b'/b values for the multi-stepped plates (n = 1 to 10). As the mesa steps increased, the b'/b values of all a/b ratios and their coupling-strength were decreased. At n > 4, the minimum coupling-strength for a/b = 25 was lower than that of a beveled value. For a/b = 20, the minimum values approached to beveled ones as step n increased. As shown in Figure 7 (b), all curves decreased slightly and the plates, which contained bigger a/b values, located in the small b'/b region. The TS-1 mode was trapped in the thick region by the energy-trapping effect and the displacement in the outer region decayed exponentially. The strength of this effect was caused by the height of b'/b proportionally. If the a/b ratio of the plate was small, it needed a stronger energy-trapping effect to decay the displacement at the plate edge, therefore, small a/b located in the lower b'/b region.

To study the TS-1 displacement against the number step n, DFTs were performed for the u2 displacement in the outer region and plotted in the Figure 8. As n increased, all curves decreased, and after n = 4, their values changed a little and were one-half that of n = 1. These results confirmed that dividing the edge into steps reduces reflection at the edge of each mesa step, and therefore TS-1 and TF mode coupling is decreased.

V. Conclusion

To study the effect of multi-stepped bi-mesa structures on the decoupling of TS and TF modes, we calculated the mode-coupling characteristics of multi-stepped bi-mesa AT-cut quartz resonators. To evaluate the mode-coupling strength, DFT was performed for the TS-1 displacement in the thick region. The results showed that increasing the number of mesa steps reduced the mode coupling between the TF and TS modes. When the mesa steps were increased to more than 4, the TF component of the TS-1 displacement were very low and was almost the same as that of beveled plates. This is because the edge height of the mesa that contributes to mode coupling was decreased. However, when the mesa height was very high, the TF component in the outer region increased. Therefore, Choosing a minimum coupling-strength point near the height of the flat plate is preferable. These results indicate that the multi-stepped bi-mesa resonator has a good mode-decoupling effect, and its characteristics approach those of the beveled resonator.

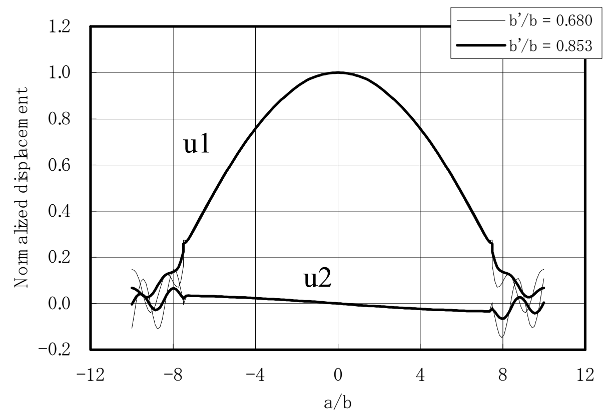
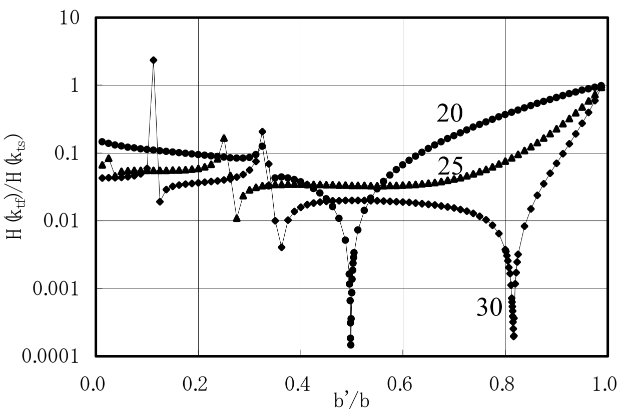
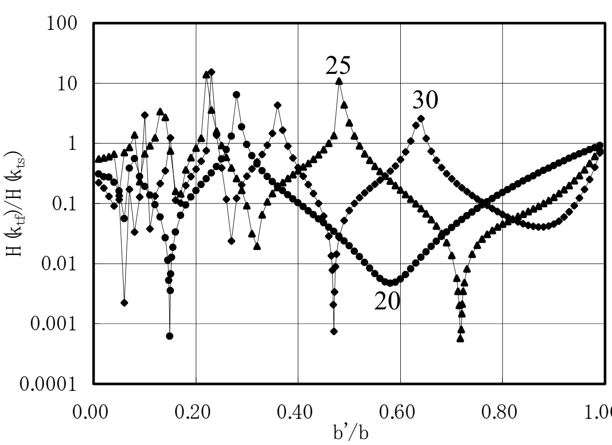


Fig. 5. TS-1 displacements of bi-mesa plates



(a) Beveled plate



(b) 3-stepped bi-mesa plate

Fig. 6. Coupling strength (Numbers indicate values of a/b)

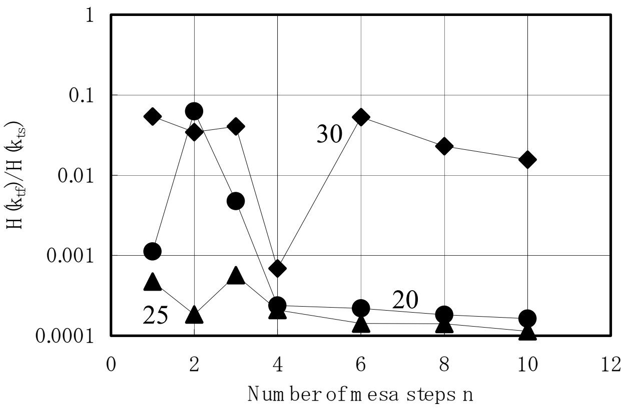


Fig. 7 (a) Minimum value of coupling strength

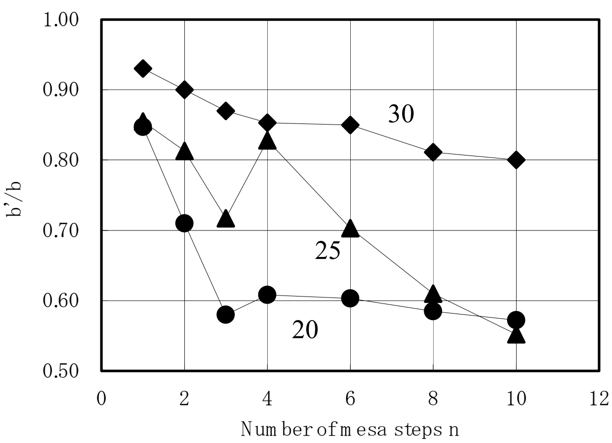


Fig. 7 (b) Minimum points of coupling strength
(Numbers indicate values of a/b)

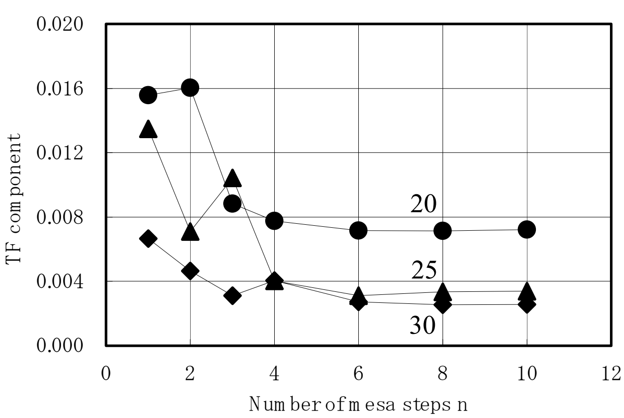


Fig. 8 TF components in outer region
(Numbers indicate values of a/b)

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